RESUME OF MATHEMATICS

1. STANDARD OF PAPERS

The Chief Examiners for Mathematics (Core) 2 and Mathematics (Elective) 2 agreed that the standard of the respective papers compared favourably with those of previous years.

2. PERFORMANCE OF CANDIDATES

While the Chief Examiner for Mathematics (Core) 2 reported that candidates' performance was not encouraging, the Chief Examiner for Mathematics (Elective) 2 noted a remarkable improvement in the performance of candidates over those of recent years

3. A SUMMARY OF CANDIDATES' STRENGTHS

(1) The Chief Examiner for Mathematics (Core) 2 listed the following as strengths of candidates:

(a) solving simultaneous equations
(b) determining sectoral angles and drawing pie charts
(c) conversion of mixed number into improper fraction
(d) calculating median and standard deviation for a given data
(e) finding the images of plane figures under given transformation
(f) deducing the identity element from a given table

(2) The Chief Examiner for Mathematics (Elective) 2 also listed the following as strengths of candidates:

(a) factorizing quadratic and cubic function
(b) using the roots of a given quadratic function to determine different equation with roots related to those of the previous one.
(c) applying the concept of Newton's 2nd Law of Motion to determine the acceleration of a body which was resting on a smooth horizontal floor
(d) finding the composite and inverse function
(e) finding the cumulative frequency of a grouped data and using the proper cumulative curve to determine percentiles (and the median) of the distribution
(f) writing down the binomial expansion of the expression $(rax)^a$ where *a* and *n* are both positive constants, and *a* is a fraction and *n* an integer.
(g) correct application of the concept of the law of conservation of linear momentum to finding the common velocity of two particles after collision
(h) correct use of this quadratic formula in solving quadratic equations.

4. A SUMMARY OF CANDIDATES' WEAKNESSES

(1) For Mathematics (Core) 2, the Chief Examiner stated that, candidates' weaknesses were shown in the following area:

(a) application of BODMAS in simplification of fractions
(b) simplification of surds
(c) circle theorems and their applications in solving problems in geometry
(d) solving problems involving trigonometry
(e) using the roots quadratic equation to determine the quadratic equation.

(2) The Chief Examiner for Mathematics (Elective) 2 also reported that candidates' weaknesses were found in the following areas:

(a) non/incorrect labelling of area
(b) candidates' inability to distinguish between the factors o a function and the roots of an equation
(c) plotting of cumulative frequencies against midpoints instead of class boundaries in drawing a cumulative frequency curve
(d) plotting frequencies against class limits instead of class boundaries in drawing histograms
(e) leaving a resultant force in a-vector-form only without magnitude, instead of magnitude and direction form
(f) leaving final numerical answers in the form of improper instead of simplifying them
(g) leaving final answers as fractions with quadratic roots in the denominators instead of rationalizing them
(h) candidates' inability to resolve forces along the sides of a figure, such as a square into column vectors
(i) poor expression for *p* and *c* and then solving for *r*.

5. SUGGESTED REMEDIES

The Chief Examiner for Mathematics (Core) 2 and Mathematics (Elective) 2 suggested that:

(i) students should be given adequate and effective teaching in concepts and skills necessary to help them overcome their weaknesses.
(ii) quizzes and tests should be given periodically to review and reinforce concepts taught.
(iii) teachers should guide students to work problems involving the above weaknesses.

MATHEMATICS (CORE) 2

1. GENERAL COMMENTS

The standard of the paper compares favourably with those of previous years.

The performance of candidates however was not encouraging.

2. A SUMMARY OF CANDIDATES' STRENGTHS

Candidates' strengths were evident in the following areas:-

(1) solving simultaneous equations

(2) determining sectoral angles and drawing of pie charts

(3) conversion of mixed number into improper fraction

(4) calculating median and standard deviation for a given data

(5) finding the images of plane figures under given transformation

(6) deducing the identity element from a given table

3. A SUMMARY OF CANDIDATES' WEAKNESSES

Candidates' weaknesses were shown in the following areas:-

(1) application of BODMAS in simplification of fractions

(2) simplication of surds

(3) circle theorems and their applications in solving problems in geometry

(4) solving problems involving trigomometry

(5) using the roots of quadratic equation to determine an equation.

(6) Understanding the concept of standard deviation and its calculation using a given frequency distribution.

4. SUGGESTED REMEDIES

Students should be given adequate and effective teaching in concepts and skills necessary to help them overcome the weaknesses.
Quizzes and tests to be given periodically to review and reinforce concepts taught.

5. DETAILED COMMENTS

Question 1

(a) Simplify without using a calculator: $\dfrac{3\frac{4}{7}-1\frac{1}{3}\div 2\frac{2}{5}}{\frac{13}{3}-1\frac{6}{7}}$

(b) Simplify: $\frac{2}{3+2\sqrt{2}} + \frac{1}{3-2\sqrt{2}}$ leaving your answer in the form $a + b\sqrt{2}$

In (a), candidates were able to express the mixed fractions into improper fractions but could not apply the concept of BODMAS to proceed with the simplification.

The part (b) was poorly done since most of the candidates could not find the LCM of the given expression and hence failed to simplify the given expression.

__Question 2__

(a) Simplify: $\frac{x+\frac{1}{x+2}}{x^2-1}$

(b) The sum of the ages of Akin and Dop is 35 years.

The sum of twice Akin's age and three times Dop's age is 89 years. Find their present ages.

The part (a) posed a big challenge to candidates who attempted it. Most of them started well by attempting to write the numerator as a single fraction but instead of getting $\frac{x^2+1+2x}{x(x^2-1)}$, most of them left the in the denominator out to get $\frac{x^2+1+2x}{x^2-1}$ instead of $\frac{x^2+1+2x}{x(x^2-1)}$

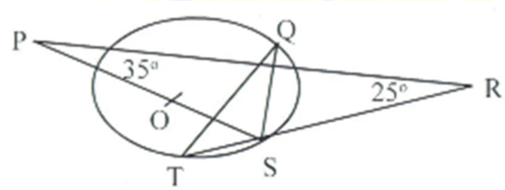
They were however able to factorize $x^2 + 1 + 2x$ to get $(x + 1)(x + 1)$ and to get $x^2 - (x + 1)(x - 1)$

Hence the given expression simplifies to $\frac{(x+1)(x+1)}{x(x+1)(x-1)} = \frac{x+1}{x(x-1)}$

The (b) part was well-handled by candidates. Correct equations were obtained and solved simultaneously to get the ages of Akin and Dop.

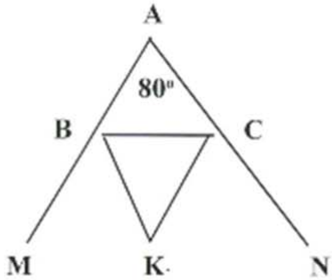
__Question 3__

(a)



I the diagram, *PQR* and *TSR* are are straight lines, $\angle ORS = 25^\circ$ and $\angle OPS = 35$. Find $\angle SQT$.

(b)



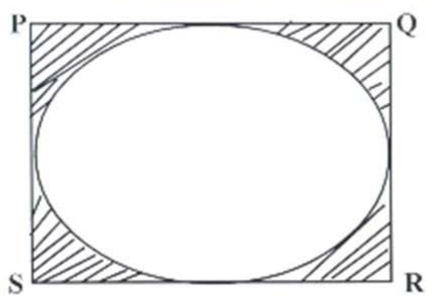
In the diagram, $\overline{AM}$ and $\overline{AM}$ are staright lines.
ABC is an isosceles triangle, $\angle BAC$ = 80, the bisectors of $\angle MBC$ and $\angle NCB$ meet at K.
Calculate $\angle BK$.

Part (a) posed a serious problem to candidates who attempted it. They could not recall the appropriate circle theorem relations to answer the question.

In (b) most candidates could not provide solution beyond the application of equality of base angles of isosceles triangles.

<u>Question 4</u>

(a)



In the diagram, a circle is drawn in a square PQRS as shown. If the total area of the shaded portion is 2cm^2, calculate the radius of the circle. $\left[Take\ \frac{22}{7}\right]$

(b) A cube of length 4cm has the same volume as a cone with base diameter 7 cm. Find, correct to the nearest cm, the height of the cone. $\left[Take\ \frac{22}{7}\right]$

Performance in (a) was not the best. Candidates were not able to recognize that the diameter of the circle is the same as the length of the square. As a result they were unable to obtain the appropriate equation to enable them solve for the radius of the circle.

In (b) a lot of candidates could not quote the formula for the volume of a cone correctly even though this could have been obtained from the formula book.

Question 5

The table shows, in percentage, the monthly expenditure of an employee whose gross monthly salary is GH¢10,800.

Item	Percentage
Social security	5
Income Tax	25
Food	40
Transport	10
Rent	12.5
Others	7.5

(a) Draw a pie chart to illustrate the data.

(b) If the social security contribution and income tax are deducted from the gross monthly salary before payment, correct to the nearest whole number, the expenditure on rent as a percentage of the employee's take-home pay.

This was a question in which performance was creditable.
In part (a) candidates calculated sectoral angles and constructed the required pie chart.

In (b) however, candidates seemed not to know what "take-home pay" is. This affected their final result.

Question 6

In a survey, 74 out of 88 tourists interviewed said they had visited at least one of Africa(A), Europe (E) and South America(S). Of these, 19 had visited Europe and Africa, 30 Europe and South America, 26 South America and Africa. No one had visited only Africa, 10 only Europe, 7 only South America and x had visited all the three continents.

(a) Draw a Venn diagram to represent this information.

(b) Write down a suitable equation in x and find the value of x.

(c) Find:

(i) $n[(E \cap A) \cup S]$;

(ii) $n(A)$;

(iii) $n[(E \cup S)' \cap A]$.

This was a very popular question. However most of the candidates could not illustrate the information properly on the Venn diagram.

Question 7

(a) If $U = 1 - \frac{3v}{vt-w}$, make t the subject of the relation.

(b) The graph is that of the relation $y = ax^2 + bx + c$.

(i) From the graph determine: $y = ax^2 + bx + c = 0$
the minimum value of y;
the roots of the equation

(ii) Using the roots, determine the value of a, b and c.

A good number of candidates attempted the question but their performance fell below expectation.
In (a), candidates were to make t the subject of relation $U = 1 - \frac{3v}{vt-w}$
In attempting to clear the fraction most candidates did not multiply "1" by $vt - w$ thus missed the correct relation.

In (b), the question demanded the reading of graph and determination of unknown variables. Most of the candidates were able to find the roots but could not write down the quadratic equation and also determine the values of a, b, and c.

Question 8

(a) From a point P on a horizontal ground, the angle of elevation of the top S of a tower (RS) 32m high is 58°.

(i) Calculate the distance PR, correct to the nearest metre.

(ii) If P is due south of the tower and another point Q is 35m due east of the tower, calculate, correct to one decimal place the:
distance PQ;
bearing of P from Q.

0	**2**	**4**	**6**	**8**
2	**4**	**8**	**2**	**6**
4	**8**	**6**	**4**	**2**
6	**2**	**4**	**6**	**8**
8	**6**	**2**	**4**	**4**

The table is a multiplication table in modulo 10 over the set Find the identity element.
Part (a) was not popular among candidates.
Most candidates who attempted it could not sketch the correct diagrams to use in solving the problem.

In (b) most of the candidates who attempted the question did not seem to have the slightest idea of the concept of identity elements. Mathematically, if e is an identity element then
$a*e = e*a = a$ for all elements a.
From the given table it is only the element 6 which has this property and so the identity element is 6.

<u>**Question 9**</u>

An aircraft leaves airport *P* and flies on a bearing 060°. From a distance of 10 km due east of *P*, another aircraft leaves airport *R* and flies on a bearing of 330°. Both planes meet at *Q*. Using ruler and a pair of compasses only, with a scale of 1 cm to 1 km,

(a) **construct triangle *PQR*.**

(b) **How far is Q from:**
  (i) ***P* ;**
  (ii) ***R?***

(c) **measure $\angle PQR$.**

(d) **Calculate how far north of *R*, *Q* is, correct to one decimal place.**

Very few candidates answered this question. Candidates were to use a pair of compasses and ruler only to construct with a given scale. Most of the candidates who attempted the questions were unable to do the construction well.
With (a), what was to be done was to construct |PR| = 10 cm with *R* due east of *P*. With a good sketch one would have known that $Q\hat{P}R$ and $P\hat{R}Q =$ 60°

<u>**Question 10**</u>

Marks	**3**	**4**	**5**	**6**	**7**	**8**
Frequency	**5**	**m**	**m+1**	**9**	**4**	**1**

The table gives the frequency distribution of marks obtained by a number of students in a test. If the mean mark is 5, calculate the:

(a) **value of m;**

(b) **median;**

(c) **standard deviation of the distribution.**

This question demanded the finding of some missing frequencies in a frequency distribution table. Most candidates could not find $\sum f(x)$ in terms of m so could not equate $\frac{\sum fx}{\sum f} = 5$ to get m.
Once the wrong value of m was calculated the median and the Standard deviation were wrong.

Question 11

(a) **Given that sin x = 0.6 and $0^o \leq x \leq 90^o$, find 1-tan x, leaving your answer in the form $\frac{a}{b}$ where a and b are integers.**

(b) **Two equal chords PQ and QR are each 12 cm long. They meet at a point Q on the circle making angle PQR .**
Calculate, correct to the nearest whole number the:
(i) **radius of the circle;**
(ii) **perimeter of the major segment cut off by the chord PR.**

In (a) candidates were to find 1 - tan x in form $\frac{a}{b}$ where a and b are integers given that Sin x = 0.6 and $0^o \leq x \leq 90^o$.
By writing 0.6 as a common fraction we get Sin x = 0.6 = $\frac{6}{10}$ = $\frac{3}{5}$ and from Pythagorean triplets the third side of a right-angled triangle, two of whose sides are 3 and 5 is 4.

Thus tan x = $\frac{3}{4}$ and therefore 1 - tan x = 1- $\frac{1}{4}$ = $\frac{3}{4}$.

Candidates had problem with the second part too. Almost all who attempted this part got it wrong.

Question 12

(a) **Find the image of (-2,4) under the mapping** $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2y \\ y - 3x \end{pmatrix}$

(b) **Two functions f and g are defined as:**

$$\mathbf{f:}\ x \rightarrow \frac{x^2}{4} - 9.$$

$$\mathbf{g:}\ x \rightarrow \frac{1}{2x}(x \neq 0).$$

(i) **Evaluate** $f(4) + g\left(\frac{-1}{3}\right)$

(ii) **If f x g = 2, solve for x.**

In (a) candidates were able to find the image of (-2,4) under the given mapping, but left the answer as a column vector.
Candidates answered the (b) very well. They evaluated $f(4) + g\left(\frac{-1}{3}\right)$ correctly. They also solved f x g = 2 correctly.

Question 13

(a) **Using a scale of 2 cm to 2 units on both axes, draw on a graph sheet two perpendicular axes OX and OY for the interval $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$.**

(b) **Draw clearly and label the vertices as appropriate:**

(i) **triangle PQR with $P(1,2)$, $Q(5,3)$ and $\overrightarrow{RQ} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$**

(ii) **the image $\Delta P'Q'R'$ of ΔPQR under a rotation of 180^{o} about the origin where $P \rightarrow P', Q \rightarrow Q'$, and**

(iii) **the image $\Delta P"Q"R"$of $\Delta P'Q'R'$ under a reflection in the line $x = 0$ where $P \rightarrow P'', Q \rightarrow Q''$, and $R \rightarrow R''$.**

(c) **Describe fully, a single transformation that maps ΔPQR onto $\Delta P"Q"R$.**
ΔPQR.

Candidates were able to find the required images under the given transformations. They also described the single transformation that mapped $\Delta P"Q"R$ unto ΔPQR

MATHEMATICS (ELECTIVE) 2

1. GENERAL COMMENTS

The standard of the paper compares favourably with that of previous years. Candidates have shown a remarkable improvement in their performance over that of recent years.

2. A SUMMARY OF CANDIDATES' STRENGTHS

Candidates' strengths were evident in the following areas;

(1) Factorizing quadratic and cubic functions.
(2) Using the roots of a given quadratic function to determine different equations with roots related to those of the previous one.
(3) Applying the concept of Newton's 2nd Law of motion to determine the acceleration of a body which is resting on a smooth horizontal floor.
(4) Finding composite and inverse functions.
(5) Finding the cumulative frequency for a grouped data and using the proper cumulative curve to determine percentiles (and the median) of the distribution.
(6) Writing down the Binomial expansion of the expression $(1 + ax)^n$ where *a* and *n* are both positive constants, and *a* is a fraction and *n* as integer.
(7) Correct application of the concept of the law of conservation of linear momentum in finding the common velocity of two particles after collision.
(8) Correct use of quadratic formula in solving quadratic equations.

3. A SUMMARY OF CANDIDATES' WEAKNESSES

Candidates' weaknesses were shown in the following areas:

(1) Non/Incorrect labelling of area.
(2) Candidates inability to distinguish between the factors of a function and the roots of an equation.
(3) Plotting cumulative frequencies against midpoints instead of class boundaries in drawing a cumulative frequency curve.
(4) Potting frequencies against class limits instead of class boundaries in drawing histogram.
(5) Leaving final answers in the form of improper fraction instead of simplifying it.
(6) Leaving final answers as fractions with quadratic surds in the denominators instead of rationalizing them.
(7) Candidates' inability to resolve forces acting along the sides of a figure, such as a square into column vectors.
(8) Poor expressions for and then solving for *r*.

4. SUGGESTED REMEDIES

(i) Teachers and candidates should take good note of the above weaknesses and improve upon teaching and learning.

(ii) Teachers should guide candidates/students to work problems involving the above weaknesses.

5. DETAILED COMMENTS

Question 1

If $f(x) = 6x^3 + px^2 + 2x - 5$ and $f(-1) = 0$, where p is a constant, find;

(a) the value of p,

(b) the factors of $f(x)$.

The question was very popular and candidates' performance was quite good.

Most of the candidates who attempted the question were able to use the Factor Theorem properly to obtain a correct quadratic equation from *f(x)*, which they factorized properly and obtained the factors of *f(x)*.

Some of the candidates worked further in equating the factors to zero and solving for *x*, which was not asked for.

Question 2

If the roots of the quadratic equation $3x^2 + 4x + 1 = 0$ are α and β, determine the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Candidates' performance was average. In the question the roots of the equation $3x^2 + 4x + 1 = 0$ were given as α and β and candidates were asked to determine the equation whose roots were $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Most of the candidates who attempted it were able to state that $\alpha + \beta = -\frac{4}{3}$ and $\alpha\beta = \frac{1}{3}$.

However, some of them expressed $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ as $\frac{(\alpha+\beta)^2 + \alpha\beta}{\alpha\beta}$ instead of $\frac{(\alpha+\beta)^2 - \alpha\beta}{\alpha\beta}$ and therefore the subsequent workings were wrong.

Question 3

(a) Write down in ascending powers of x, the first four terms of the expansion of $(1 + 0.5x)^6$.

(b) using your answers in 3(a), evaluate correct to four decimal places $(1.02)^6$.

Candidates' performance was satisfactory.

In part 3(a), the expansion work was satisfactorily done, there were however some faulty algebraic manipulations.

In part 3(b), most of the candidates were able to deduce the value of x as 0.04, but quite a number of the candidates replaced x with 0.02 without comparing $(1 + 0.5x)$ with (1.02), hence the wrong solution obtained.

Question 4

Solve: $6(9^x) + 3^x - 2 = 0$.

Candidates' performance was fairly good.

Most of the candidates who attempted it were able to express the LHS as $6a^2 + a - 2$ and solve for a. However, quite a number of them did not continue to solve for x, and were penalized.

Question 5

Write down the following vectors in the form ($ai + bj$) where a and b are scalars.

(a) (i) $m = (N, 180^o)$
(ii) $n = (10N, 150^o)$

(b) Using your results in 5(a), find ($2m + 3n$).

The performance of the candidates who attempted this question was average.

Candidates had difficulty expressing the vector in the required form; a lot of them presented it as $m = (0i + -4j)$ *or* $m = (-4j)$, instead of $m = (0i - 4j)$ but these presentations were rejected.

Candidates' performance in 5(b) was not satisfactory since most of them could not find m and n in (a) i and ii.

Question 6

If ${}^6P_n = 24\,({}^6C_n)$, find the value of n.

The question was not very popular and candidates' performance was not good enough. Candidates were to find the value of n if ${}^6P_n = 24\,({}^6C_n)$.

Only few of the candidates who attempted the question were able to find the value of n correctly; a lot of them were unable to state the correct expansion of ${}^6P_n$ and ${}^6C_n$, and therefore were unable to find the correct value of n.

Some of them who were able to state the correct expansion later got everything messed up in the process when finding the value of n.

A solution is presented below:

$$^6P_n = 24(^6C_n)$$

$$\frac{6!}{(6-n)!} = 24\left(\frac{6!}{(6-n)!n!}\right)$$

$$\frac{n!6!}{(6-n)!} = 24\left(\frac{6!}{(6-n)!}\right)$$

$$n! = 24$$

$$n = 4$$

Question 7

The table below shows the distribution of the monthly income of 50 workers in a company.

Monthly Income ***(in thousand GH¢)***	**120-129**	**130 -139**	**140- 149**	**150 - 159**	**160 - 169**
No. of workers	**5**	**16**	**9**	**14**	**6**

(a) **Draw a histogram for the distribution.**

(b) **If one worker is selected at random, what is the probability that he earns at most GH¢144,000.00?**

Candidates' performance was average.
The histogram was quite satisfactorily drawn.

Some candidates were penalized for plotting frequencies against class limits, non labelling of axes and wrong labelling of axes . Midpoint values were also located at the boundaries, which attracted very severe penalties.

Finding the probability seemed to be a problem for the candidates. A lot of them were dividing the maximum amount earned by the number of workers instead of dividing the number of workers who earned up to that amount by the total number of workers.

Question 8

A body of mass 800g resting on a smooth horizontal floor is acted upon by three forces; *F1* = (18*N*, 330°), *F2* = (10*N*, 90°) and *F3* = (25*N*, 180°),
Find;

(a) The resultant force correct to one decimal place.

(b) Its initial acceleration.

Candidates' performance was satisfactory.

The resolution of the forces into vertical components were correctly done by most of the students. Finding the magnitude of the resultant force was satisfactorily done. However, some of the students just ended at finding the magnitude instead of finding the direction as well as vector, since the resultant force has both magnitude and direction.

In fact, most of the candidates interchanged the components and therefore, had the resultant components wrong, but it did not affect the computation of the magnitude.

In finding the acceleration, most of the candidates used Newton's 2nd Law of Motion properly.

<u>Question 9</u>

(a) Evaluate:

(b) Find from first principles the derivative of
$y = \left(3x^4 + x^3\right) x^{-1}$ where x 0.

Candidates' performance was average.
Most of the candidates who attempted the question 9(a) saw the need to expand the expression before integrating it. After the expansion the integration was properly carried out in most cases. Others integrated the two terms as they appeared in the question but that was a wrong approach.

In 9(b) candidates were to find from first principles the derivative of $y = (3x^4 + x^3)\, x^{-1}$ where x 0.
Some of the candidates expanded the R.H.S. properly and had it easier finding the derivative, than the candidates who wrote the R.H.S. as or those who started finding the derivative of the expression as it was given; the manipulation became more tedious and increased the probability of faulting.

<u>Question 10</u>

Functions *f* and *g* are defined on the set of real numbers *R*, by

$f: x \rightarrow 3 - x^2$

$g: x \rightarrow \sqrt{2(x-1)}$

Find:

(a) $f \circ g$,

(b) f,

(c) the values of x, correct to two decimal places for which f $o\ g\ (x) = g(x\text{-}1)$.

Many candidates answered this question and performance was quite good.

Question 10(a) and 10(b) were solved by most of the candidates, few of them however, equated their results to zero and went ahead to find the values of x which were not stated in the question.
Most of the candidates could not solve 10(c) since they had difficulty in finding $g(x - 1)$.

Question 11

Two linear transformations P and Q of the x - y plane are given by $P(x,y)=(3x - 2y, x - 5y)$ and $Q(x,y)=(-2x + y, 3x - y)$.

(a) Write down the matrices of the transformations.

(b) Find the matrix R such that $PQ + R = P + Q$.

(c) Find the image of $B(5, 6)$ and $C(4,9)$ under the transformation $(2P + 3Q)$.

(d) Find the equation of the line joining B' and C' where B' and C' are the images of B and C, respectively.

The question was popular and candidates' performance was good.

In part 11(a) candidates were asked to write down the matrices of the transformations.

Most of the candidates solved this question without difficulty but a few of them however mixed up the entries.
In part 11(b) candidates were to find the matrix of R such that $PQ + R = P + Q$.

This part was well handled by most of the candidates but a few of them could not solve the algebraic aspect of the question..

In 11(c) the image of the points $B(5,6)$ and $C(4,9)$ were to be found under the transformation $(2P + 3Q)$.
Finding the transformation $(2P + 3Q)$ seemed to be quite manageable, but in finding the image, however, some of the candidates used wrong approach such as to obtain (-6, 23) which was totally unacceptable.

In 11(d) candidates were to find the equation of the line joining B' and C'.
This was satisfactorily handled by most of the candidates who attempted it.

Question 12

(a) If $x^2 + y^2 = py(1 + x^2)$, where p is a constant, find .

(b) The curve $y = ax^2 + bx - 3$ passes through point $P(1, -5)$. If its gradient at P is 2, find,
(i) the values of a and b;
(ii) the minimum value of y.

The question was not popular. However, candidates' performance was quite good.

Question 12(a) was quite well-handled by the few candidates who attempted it. A few of them expanded the R.H.S. which made the differentiation easier to handle.

Most of the candidates who answered question 12 (b)(i) made the right use of the pieces of information given and got the appropriate equations involving a and b, which they solved simultaneously for the values of a and b. There were, however, a few faulty algebraic manipulations.

In 12(b) (ii) most of the candidates seemed to be avoiding the use of completing-the-square method in finding the minimum value of y, they rather chose the calculus approach which confused them and as a result could not find the minimum value of y.
Presented below are the two methods of finding the minimum value of y:

COMPLETING THE SQUARE

$$y = 4x^2 - 6x - 3$$
$$= 4\left(x^2 - \frac{3}{2}x - \frac{3}{4}\right)$$
$$= 4\left[\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{4}\right)^2 - \frac{3}{4}\right]$$
$$= 4\left(x - \frac{3}{4}\right)^2 - \frac{9}{4} - 3$$
$$= 4\left(x - \frac{3}{4}\right)^2 - \frac{21}{4}$$

Minimum value of $y = \frac{21}{4}$

CALCULUS METHOD

$$y = 4x2 - 6x - 3$$
$$\frac{dy}{dx} = 8x - 6$$

At the minimum point

$$\therefore 8x - 6 = 0$$
$$x = \frac{3}{4}$$
$$y = 4\left(\frac{3}{4}\right)^2 - 6\left(\frac{3}{4}\right) - 3$$
$$= \frac{9}{4} - \frac{9}{2} - 3$$
$$= -\frac{21}{4}$$

Minimum value of $y = \frac{21}{4}$

Question 13

In a community, 10% of the people tested positive to the HIV virus. If 6 persons from the community are selected at random, calculate, correct to four decimal places the probability that;

(a) exactly 5,
(b) none,
(c) at most 2, tested positive to the virus.

The question was not popular. However the few candidates who attempted it performed quite well.
Gradually candidates are becoming more conversant and confident in solving problems on Probability involving binomial distribution.

Question 14

The table gives the distribution of marks obtained by some students in an examination.

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	4	6	10	10	13	17	20	15	4	1

(a) Draw a cumulative frequency curve of the distribution.
(b) From your curve, estimate;
(i) the 80th percentile,
(ii) the median,
(iii) the inter-quartile range.

The question was very popular and candidates' performance was quite good.
In the question candidates were given a frequency distribution table of the marks obtained by some students in an examination and were asked to draw a cumulative frequency curve of the
distribution.

Most of them were able to draw the proper curve.

However, some of them plotted the cumulative frequencies against midpoint values which was unacceptable, rather the cumulative frequencies should be plotted against the appropriate class distribution.

The 80th percentile and the median were properly estimated by most of the candidates. Also, candidates had difficulty estimating the interquartile range. They were unable to read the upper quartiles and lower quartiles properly, hence they could not find the difference between the quartiles.

Another wrong approach they used was that they found the difference between the cumulative frequencies of the two quartiles and then read the mark corresponding to the frequency as the interquartile range. Some candidates also took the difference in the cumulative frequencies of the two quartiles as the interquartile range which was also wrong.

Question 15

(a) **A bag contains 5 white, 6 red and 7 black identical balls. Two balls are drawn, one after the other at random without replacement. Calculate, correct to two decimal places the probability that;**

(i) **the first ball is white and the second black,**
(ii) **second ball is white.**

(b) **A player tosses a fair coin. If it falls head he wins GH¢50.00 and if a tail he loses GH¢50.00. If a player with GH¢400.00 tosses the coin six times, calculate, the probability that he will be left with GH¢300.00.**

Candidates' performance in Question 15(a) was just average.
Candidates were penalized for not leaving the final answer in 2 decimal places.

Question 15 (b) was poorly handled by most of the candidates who attempted it. It was quite a challenging question for the candidates.

A solution is presented below:
Let x be the no. of times a head appears. Then

$$400 + 50x - 50(6-x) = 300$$
$$100x = 600 - 400$$
$$= 200$$
$$x = 2$$

$$P(H) = \frac{1}{2} \qquad P(T) = \frac{1}{2},\ n = 6,\ x = 2$$

$$P(x = 2) = {}^{6}C_{2}(0.5)^{2}(0.5)^{4}$$

$$= 15(0.25)(0.0625) \text{ or } 15 \times \frac{1}{4} \times \frac{1}{16}$$

$$= 0.2344 \text{ or } \frac{15}{64}$$

Question 16

(a) **Forces $\sqrt[5]{5}$N, $\sqrt[5]{3}$N, $\sqrt{5}$N and $\sqrt[3]{3}$N act along the sides $\overrightarrow{AB}$, $\overrightarrow{CB}$, and $\overrightarrow{AD}$ respectively of a square *ABCD*. Calculate the magnitude of the resultant force.**

(b) **A particle of mass 6 kg moves with a velocity ms^{-1}. and collides with another particle of mass 4 kg moving with a velocity $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ ms^{-1}. If after impact they move**

together, find correct to two decimal places, the magnitude of their correct velocity $\begin{pmatrix}6\\12\end{pmatrix}$.

The question was answered by few candidates who seemed to have been lured by 16(b), as their performance was only satisfactory in 16(b) and woefully poor in 16(a).

In 16(a) forces $\sqrt[5]{5}$N, $\sqrt[5]{3}$N, $\sqrt{5}$N and $\sqrt[3]{3}$N act along the sides $\overrightarrow{AB}$, $\overrightarrow{CB}$, and $\overrightarrow{AD}$ respectively of a square *ABCD*. Candidates were asked to find the magnitude of the resultant force.

Most of the candidates who attempted this sub-question simply found the algebraic sum of the forces as though they were acting along the same line and direction.

Most of them obtained $\sqrt[6]{5}$N + $\sqrt[8]{3}$N as the resultant force and further found the magnitude of the resultant force as *IRF* = $\sqrt{\left(6\sqrt{5}\right)^2+\left(8\sqrt{3}\right)}$, which was totally wrong; forces must be added in a vector form.

The required solution is presented as follows:

$$R_F = \begin{pmatrix}3\sqrt{3}Sin\ 90^o\\3\sqrt{3}\ Cos\ 90^o\end{pmatrix} + \begin{pmatrix}\sqrt{5}\ Sin\ 0^o\\\sqrt{5}\ Cos\ 0^o\end{pmatrix} + \begin{pmatrix}5\sqrt{3}\ Sin\ 270^o\\5\ \sqrt{3}\ Cos\ 270^o\end{pmatrix} + \begin{pmatrix}5\sqrt{5}\ \ Sin\ 180^o\\5\sqrt{5}\ \ Cos\ 80^o\end{pmatrix}$$

$$= \begin{pmatrix}3\sqrt{3}\\0\end{pmatrix} + \begin{pmatrix}0\\\sqrt{5}\end{pmatrix} + \begin{pmatrix}5\sqrt{3}\\0\end{pmatrix} + \begin{pmatrix}0\\-5\sqrt{5}\end{pmatrix}$$

$$= \begin{pmatrix}-2\sqrt{3}\\-4\sqrt{5}\end{pmatrix}$$

$$R_F = \sqrt{\left(2\sqrt{3}\right)^2+\left(4\sqrt{5}\right)^2}$$

$$= \sqrt{92}$$

$$= 9.59\ N \text{ or } 2\sqrt{23}\ N$$

Question 17

(a) **If *p* = $\begin{pmatrix}5\\6\end{pmatrix}$, *q* = $\begin{pmatrix}-1\\0\end{pmatrix}$ and *r* = $\begin{pmatrix}5\\-2\end{pmatrix}$, find the values of the constants *x* and *y* such that 2*p* = 5*xq*-3*yr*.**

(b) **The position vectors m, n, r of three points *M, N, R*, respectively, are given by *m* = *i* + *j*, *n* = 2*i* + 3*j*,**
$r = 3n + 2j$.
Find:
(i) in column vector form, the unit vectors parallel to $\overrightarrow{MN}$ and $\overrightarrow{MR}$;
(ii) correct to the nearest degree, the angle between $\overrightarrow{MR}$ and $\overrightarrow{MN}$.

The question was attempted by a large number of candidates and their performance was fairly good.

Candidates were able to substitute the values of *p*, *q* and *r* properly into the given equation, obtaining two equations in x and y which were solved simultaneously for their values. However, a few errors were identified when they were solving the simultatanous equations.

In part 17(b) the position vectors *m*, *n*, *r* of the points *M*, *N*, *R* , were given by $\boldsymbol{m} = \boldsymbol{i} + \boldsymbol{j}$, $\boldsymbol{n} = 2\boldsymbol{i} + 3\boldsymbol{j}$, $\boldsymbol{r} = -3\boldsymbol{i} + 2\boldsymbol{j}$.

Most of the candidates who attempted the question were able to express the vectors in columnar form, however, in finding the unit vectors, they failed to rationalize the denominators in $\frac{1}{|\overrightarrow{MN}|}$ and $\frac{1}{|\overrightarrow{MR}|}$ respectively.

In part 17(b)(ii) most of the candidates were able to apply the concept of the dot product properly in finding the angle between the two vectors.

Question 18

(a) **A body of mass 10kg, moving in a straight line at 5ms^{-1}, is acted upon by a force of magnitude 60N at point *P* for 3 seconds. The body comes to rest at point *Q*, after another 10 seconds. Find:**
- **(i)** **the speed of the body at 3 seconds,**
- **(ii)** **the magnitude of its acceleration,**
- **(iii)** **the magnitude of its retardation,**
- **(iv)** **the distance between *P* and *Q*.**

(b) **Two forces $F_1 = 3i - 4j$ and $F_2 = -14i - 3j$ act on a particle.**
Find the third force, F_3 that will keep the particle in equilibrium.

Most of the candidates did not attempt this question, but the few who did performed averagely.

The majority of candidates who attempted this question, used Newtons 2nd Law of Motion i.e. $v = u + at$ to determine the acceleration and then used it to find the speed of the body at 3 seconds, as stated. Again, only few candidates used the concept of impulse, i.e. $Ft = m(v - u)$ to find the required speed.

In Question 18(a)(ii) since most of the candidates had found the acceleration in 18(a)(i) they only quoted the value from that sub-question. Some of the candidates were confused and could not answer the question correctly.

In Question 18(a)(iii) most of the candidates used 3 seconds as the total time spent for the retardation instead of 10 seconds.

In Question 18(a)(iv) most of the candidates found only the distance covered during the retardation, but the actual distance between *P* and *Q* is the actual sum of the acceleration period for the 3 seconds, and the retardation period of 10 seconds.

Question 18(b) was fairly well-answered. In the question two forces $F_1 = 3i - 4j$ and $F_2 = -14i - 3j$ act on a particle and candidates were asked to find the 3rd force, F_3 that will keep the particle in equilibrium.

Most of the candidates were finding the resultant of the two forces instead of the equilibrium which is negative of the resultant force, .i.e:

$$F_1 + F_2 + F_3 = 0$$

$$\begin{aligned} F_3 &= -(F_1 + F_2) \\ &= -[(3i - 4j) + (-14i - 3j)] \\ &= 11i + 7j . \end{aligned}$$